XVI. *Propoſal of a Method for meaſuring Degrees of Longitude upon Parallels of the Æquator, by* J. Michell, *B. D. F. R. S.*

Read May 8, 1766.

THERE have been already ſeveral attempts made towards diſcovering the figure of the earth, by meaſuring the length of a degree of the meridian in different latitudes: now if theſe meaſures had been ſufficiently accurate and numerous, and we could alſo depend upon the uniformity of the earth's ſurface, we might then immediately diſcover from them the form ſought; but theſe meaſures, not agreing exactly to any certain rule, leave us ſtill in ſome degree at a loſs. It is therefore much to be wiſhed, that more meaſurements of degrees upon the meridian were to be made, in order to determine with greater accuracy a queſtion of this importance.

But what would tend yet more to determine this matter, would be the meaſurement of degrees of longitude, as well as thoſe of latitude. Aſtronomers have indeed expreſſed their wiſhes that this might be done; and though no attempt has been hitherto made towards it, yet as it is probable, that ſuch meaſurements may ſome time or other take place, it will not be amiſs to ſuggeſt a method, which will admit of more exactneſs than any I have ſeen propoſed for this purpoſe, all of which, depending upon an obſervation of the time, are therefore liable to an error of fifteen ſeconds of a degree for every ſecond of time; but the method, I mean to recommend, ſtands upon the ſame foundation with the

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meaſurement of a degree of the meridian, and, the inſtruments being equally good, and the number of miles to be meaſured the ſame, the exactneſs of it, to that of a degree of the meridian, will be in the proportion of the ſine of the latitude to the radius very nearly.

In TAB. V. Fig. 7. let AB repreſent the æquator; P the pole; DLEF a parallel of the æquator; PEC a meridian paſſing through the ſtation E; PLMN a meridian paſſing through another ſtation M; and let AMEB be a great circle cutting the meridian PEC at right angles in the point E.

Then in the ſpherical triangle AMN, right angled at N, we ſhall have R: Coſ. AM [Sin. ME] : : Tan. MAN : Co-Tan. AMN: hence $\frac{\text{Tan. MAN}}{\text{R}} \times$ Sin. ME=Co-Tan. AMN; but Tan. MAN, being the Tangent of the latitude of the given place E, and therefore given, the quantity $\frac{\text{Tan. MAN}}{\text{R}}$ will likewiſe be given, and greater or leſs than unity in the proportion of the Tan. of the latitude to the R. The Co-Tan. therefore of the angle AMN, that is the Tan. of the complement of the angle AMN to 90°. will be greater or leſs than the Sine of the Arc ME, in the proportion of the Tan. of the latitude of the place, to the R. And conſequently, whilſt the Arc ME is ſmall (in which caſe the Sine, Arc, and Tangent differ very little from each other) the angular deviation of the interſection of the meridian PLMN with the great circle AMEB, from a right angle, will contain more or fewer degrees, &c. than the Arc ME nearly in the (ſame) proportion of the tan. of the latitude of the place to the R.

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By this means then, the latitude of the place and the angle PME (contained between the meridian PMN and the great circle AMB) being given, the length of the Arc ME will likewiſe be given, with great exactneſs. But I muſt obſerve, that as the angles PEM and PME muſt be taken by the obſervation of ſome ſtar near the pole, they will be leſs accurate, when reduced to the plain of the horizon, than at the pole, in the proportion of the Sine of the diſtance between the pole and zenith, that is the Coſ. of the latitude, to the R, which with the proportion juſt mentioned of the Tan. of the latitude to the R, makes the accuracy of this method upon the whole, when compared with that of the meaſurement of a degree of the meridian, in the proportion of the Tan. multiplied into the Coſ. of the latitude, to the ſquare of the R. very nearly; but the Tan. of any angle into its Coſ. is equal to the Sin. into the R. whence this proportion is the ſame as the Sin. into the R. to the ſquare of the R. and dividing both by the R. ſimply as the Sin. of the latitude, to the R. as above.

Having got the length of the arc ME, of a great circle, in degrees, &c. together with the diſtance of the two ſtations M and E, it is eaſy to conclude from thence the length of a degree of the parallel of latitude, at the place of obſervation, which will be the ſame, without ſenſible error, as it would be, ſuppoſing the earth was an exact ſphere, to the ſame ſcale, with the degree of a great circle juſt found.

For in Fig. 8. let APB repreſent a ſection of the earth through its axis PCH; ACB an æquatorial diameter; AD the radius of curvature at the point A; and PH the radius of curvature at the

 point

point P; DFH the evolute of the curve AEP; EF the radius of curvature at the point E; (which we will ſuppoſe to have the ſame latitude with the point E in Fig. 7.) and let EF be produced till it cuts the axis PH in G: then with the radius EG and centre G, deſcribe the arc IEK, which will be the leaſt circle, that can touch the curve AEP at the point E, without cutting it. Let now the curve PEA, the line EG, and the arc IEK revolve about PH as an axis, and, PE being equivalent to PE in the former figure, the point E in the latter figure will deſcribe the parallel DEF in the former; AEP at the ſame time deſcribing the ſurface of the earth, and IK deſcribing a portion of a ſphere, which will be every where a tangent to the parallel DEF, and whoſe centre will be G. The curvature therefore of this ſphere will be leſs than the curvature of the earth, in the direction of the meridian, at the point E, as the radius GE is greater than the radius FE; but this, in moderate diſtances, can cauſe no ſenſible error. The difference between AD the radius of curvature, at the point A, on the earth's ſurface, and the line AC, according to that hypotheſis, which makes it the greateſt, does not exceed one ſixtieth part of the whole, and upon the ſame hypotheſis, the part FG of the line EG, ſuppoſing E to be in the latitude of 45°, would not exceed $\frac{1}{130}$ part of the whole. If then we take any other point upon the ſurface of the earth as M, at a ſmall diſtance from E, the diſtance between that point and the ſphere deſcribed by the arc IK, will be only $\frac{1}{130}$ part of the verſed ſine of the arc EM, and the perpendicular ſtanding upon the ſurface of the earth at M, will be inclined to the perpendicular

ſtanding

ſtanding upon the ſphere in an angle, which is equal to $\frac{1}{130}$ part of the angle ſubtended by the arc EM. And in higher latitudes theſe quantities will be ſtill leſs. Let us now return to Fig. VII. and ſuppoſing the point E to be ſituated in latitude 45°, let the arc EM cutting PE at right angles, conſiſt of 2° (near 140 ſtatute miles) then will the ſide PM, of the triangle PME, conſiſt of 45° 2′ 5″$\frac{1}{2}$, and conſequently, if LM in Fig. 7. be ſuppoſed to correſpond to EM in Fig. 8. the diſtance of theſe two points E and M, in the latter Fig. will be only 2′ 5″$\frac{1}{2}$, the $\frac{1}{130}$ part of the verſed ſine of which is a little more than $\frac{1}{3}$ of an inch, to the radius of the earth, which will therefore be the diſtance of the point M upon the earth's ſurface, and the point of the imaginary ſphere, deſcribed by IK, immediately over it. Hence alſo, the inclination of the real perpendicular at M, and the imaginary one ſtanding upon the arc IK, at the ſame place, to each other, will be ſomething leſs than a ſecond, a quantity in itſelf almoſt too ſmall to be regarded, unleſs the inſtruments made uſe of are both very large and very excellent in their kinds, and which, being wholly in the plane of the meridian, will produce an error, that muſt be perfectly inſenſible, with any inſtruments whatſoever, in an obſervation of the angle PME, Fig. 7. which will therefore, to all intents and purpoſes, be the ſame, as if the curvature of the earth in the direction of the meridian, and in the direction of ME or LE were accurately the ſame.

I have ſuppoſed the arc ME, to ſtand at right angles to the meridian PE, which paſſes through one of the extream ſtations; the method here propoſed is, however, liable to the leaſt error, when the meridian cuts the arc to be meaſured at right angles in the middle of it; but this makes ſo very ſmall a difference, that it is not worth regarding; nor is it indeed neceſſary, that the arc ſhould not deviate two or three degrees from right angles with the meridian, at that end where it cuts it moſt nearly at right angles, in caſe the ſituation and circumſtances of the country ſhould make this more convenient, the errors, that would be occaſioned by ſuch a deviation, being too ſmall to affect the concluſion. And if this deviation was ſtill much greater, and the length of a degree of the meridian at the ſame place was known, it would be very eaſy to make the neceſſary corrections.

It will perhaps be objected, that the method above propoſed depends, in ſome meaſure, upon time, as well as others, the finding of the meridian not being to be performed without it; but I muſt obſerve, that the motion of the pole ſtar, by which I propoſe to find the meridian, being ſlower than that of a ſtar at the æquator, nearly in the proportion of 30 to 1, this method will admit of an exactneſs greater in the ſame proportion (except the reduction of the Sin. to the R. before mentioned) than thoſe obſervations, by which we endeavour to find the difference of the longitude of two places, by the difference of the time of the ſun, or a ſtar's coming to their reſpective meridians.

The method above propoſed will likewiſe require different inſtruments from thoſe commonly in uſe; but admitting, that inſtruments of equal radius are capable of equal exactneſs, this method will admit of the ſame exactneſs with the obſervations of a degree of the meridian, except the before-mentioned limitation.

tion. Nor would the inſtruments for this purpoſe, if well contrived, be either leſs portable, or more expenſive, than thoſe for meaſuring a degree on the meridian; the ſame teleſcope which would be neceſſary for finding the meridian, would ſerve likewiſe for tracing the arc of a great circle; but it is beſide my preſent purpoſe to conſider any farther what kind of inſtruments would be moſt proper upon this occaſion, or what would be the beſt manner of conſtructing them.

I muſt not, however, diſmiſs the preſent ſubject, without obſerving, that, by means of the above-mentioned method, a country not too near the æquator, nor attended with any other unfavourable circumſtances, might be laid down with wonderful exactneſs. By running a great circle nearly Eaſt and Weſt through the midſt of it, we ſhould get the longitude of all the places, the great circle paſſed over; and if, by means of the meridian teleſcope, we ſhould trace meridians through a few of theſe places, as far North and South, as the ſurvey was intended to be carried, we ſhould then have a number of ſtations, in ſeveral parts of the country, whoſe longitudes, with reſpect to one another, would be very accurately determined, and to which other places might eaſily be referred, when the length of a degree of longitude in thoſe ſituations was known.

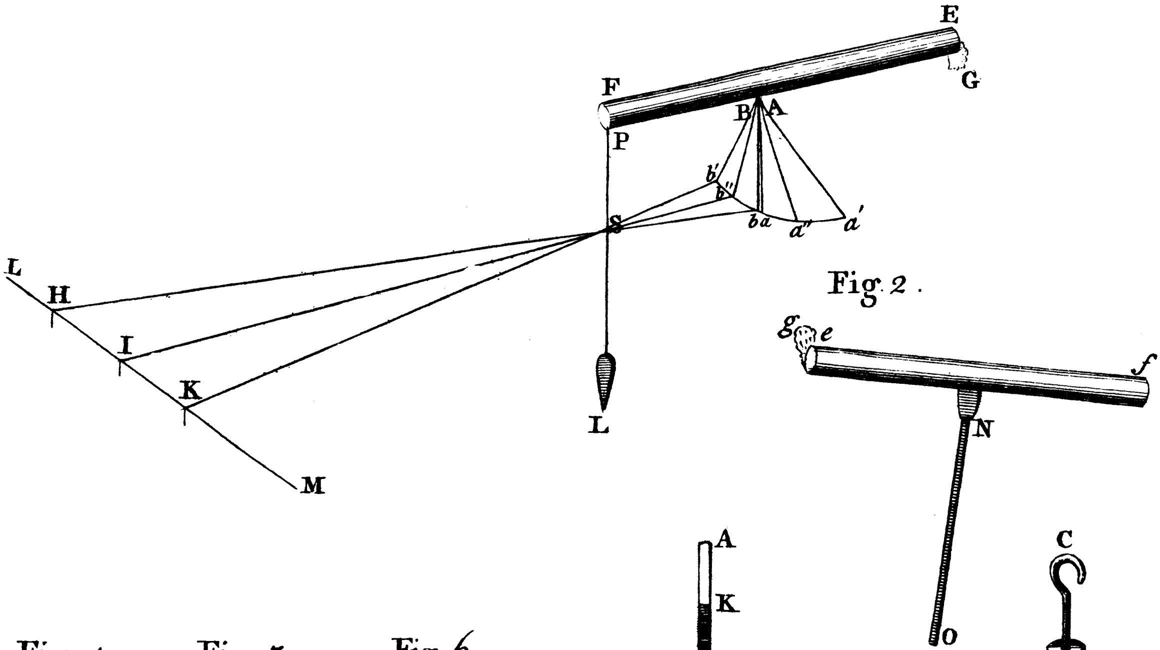


Fig. 2.

Fig. 4.

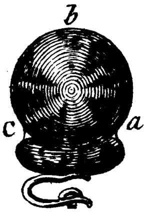


Fig. 5.

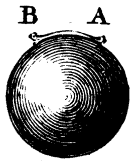
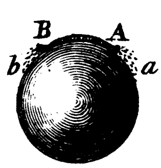


Fig. 6.



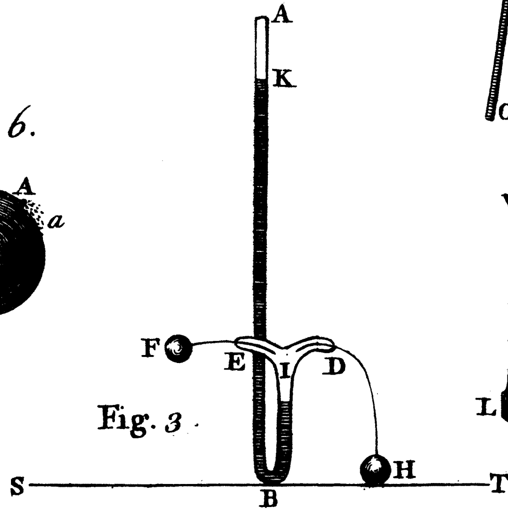


Fig. 3.

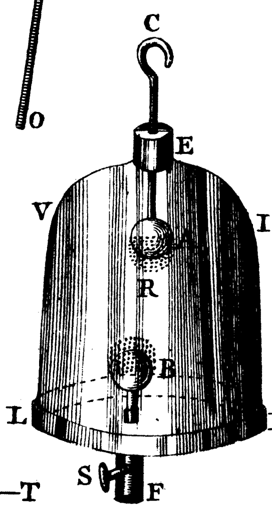


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Fig. 7.

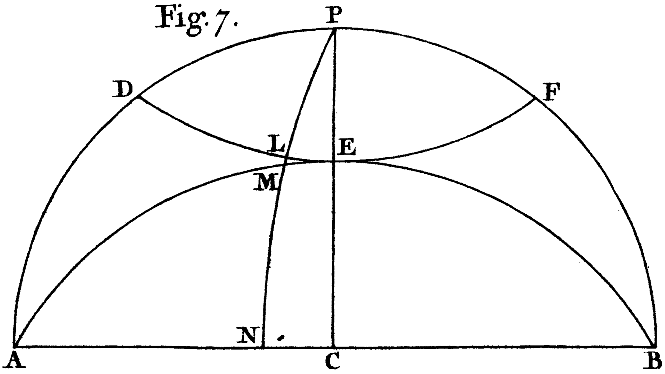


Fig. 8.

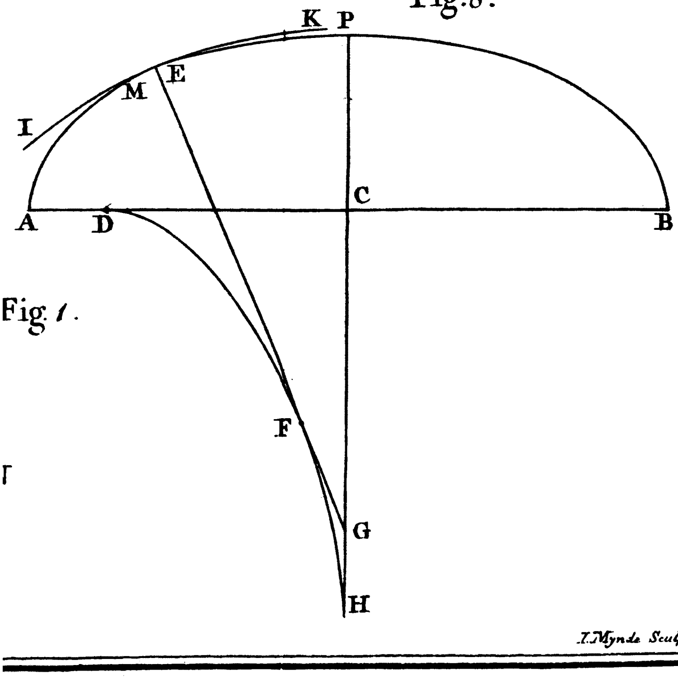
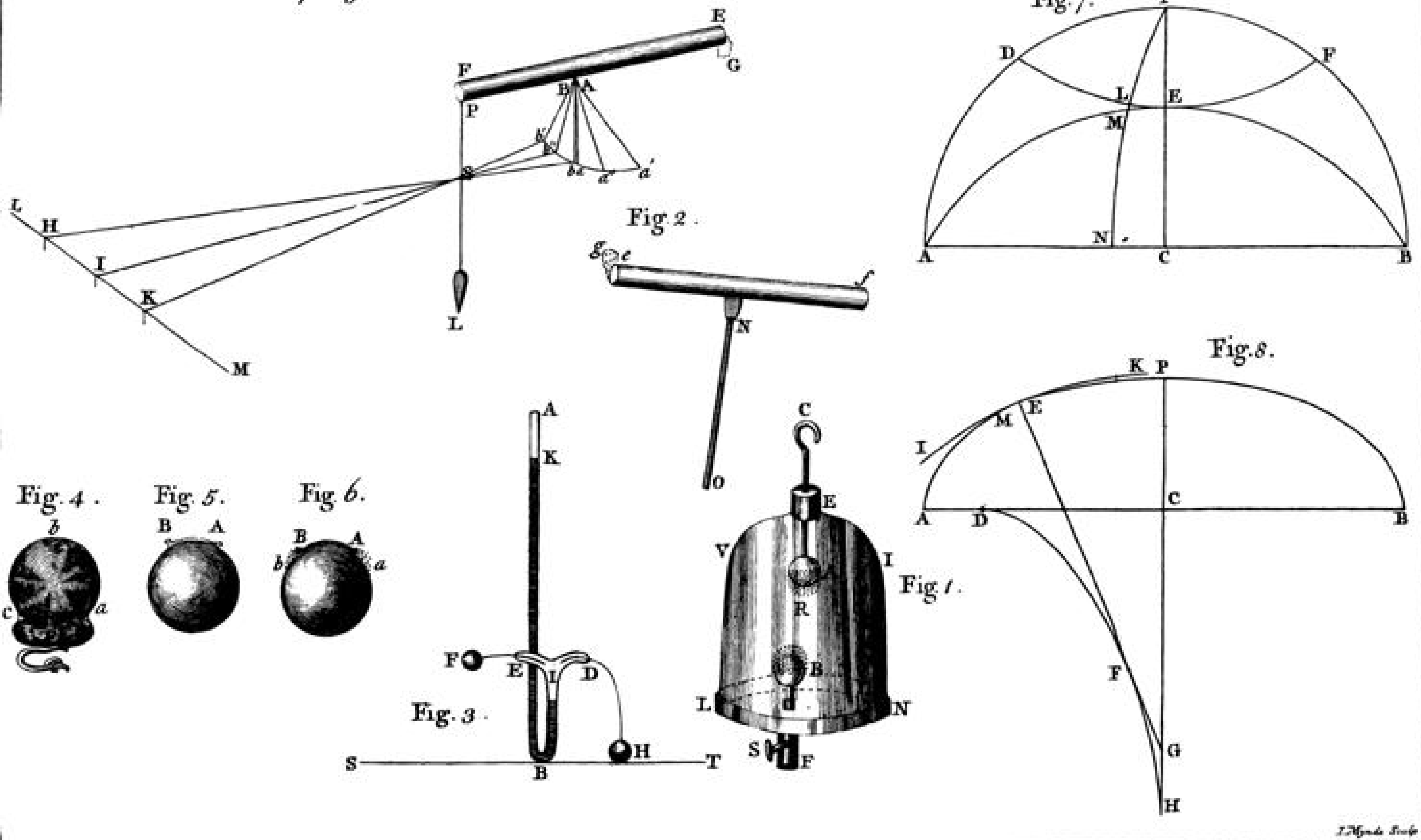


Fig. 1.

J. Mynde Sculp



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